

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number	Candidate Number		

Pearson Edexcel Level 3 GCE

Thursday 25 May 2023

Afternoon (Time: 1 hour 30 minutes)	Paper reference	9FM0/01
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Further Mathematics

Advanced

PAPER 1: Core Pure Mathematics 1

You must have: Mathematical Formulae and Statistical Tables (Green), calculator	Total Marks
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Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. The cubic equation

$$x^3 - 7x^2 - 12x + 6 = 0$$

has roots α , β and γ .

Without solving the equation, determine a cubic equation whose roots are $(\alpha + 2)$, $(\beta + 2)$ and $(\gamma + 2)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p , q and r are integers to be found.

(5)

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Question 1 continued

(Total for Question 1 is 5 marks)

2. (a) Write $x^2 + 4x - 5$ in the form $(x + p)^2 + q$ where p and q are integers.

(1)

(b) Hence use a standard integral from the formula book to find

$$\int \frac{1}{\sqrt{x^2 + 4x - 5}} dx$$

(2)

(c) Determine the mean value of the function

$$f(x) = \frac{1}{\sqrt{x^2 + 4x - 5}} \quad 3 \leq x \leq 13$$

giving your answer in the form $A \ln B$ where A and B are constants in simplest form.

(3)

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Question 2 continued

(Total for Question 2 is 6 marks)

3.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$z_1 = -4 + 4i$$

- (a) Express z_1 in the form $r(\cos \theta + i \sin \theta)$, where $r \in \mathbb{R}$, $r > 0$ and $0 \leq \theta < 2\pi$ (2)

$$z_2 = 3 \left(\cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12} \right)$$

- (b) Determine in the form $a + ib$, where a and b are exact real numbers,

(i) $\frac{z_1}{z_2}$ (2)

(ii) $(z_2)^4$ (2)

- (c) Show on a single Argand diagram

- (i) the complex numbers z_1 , z_2 and $\frac{z_1}{z_2}$
- (ii) the region defined by $\{z \in \mathbb{C} : |z - z_1| < |z - z_2|\}$ (4)

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Question 3 continued



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Question 3 continued

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(Total for Question 3 is 10 marks)



4. Prove by induction that for $n \in \mathbb{N}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & -2n \\ 0 & 1 \end{pmatrix}$$

(5)

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Question 4 continued

(Total for Question 4 is 5 marks)

5. The line l_1 has equation $\frac{x+5}{1} = \frac{y+4}{-3} = \frac{z-3}{5}$

The plane Π_1 has equation $2x + 3y - 2z = 6$

(a) Find the point of intersection of l_1 and Π_1

(2)

The line l_2 is the reflection of the line l_1 in the plane Π_1

(b) Show that a vector equation for the line l_2 is

$$\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

where μ is a scalar parameter.

(5)

The plane Π_2 contains the line l_1 and the line l_2

(c) Determine a vector equation for the line of intersection of Π_1 and Π_2

(2)

The plane Π_3 has equation $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = b$ where a and b are constants.

Given that the planes Π_1 , Π_2 and Π_3 form a sheaf,

(d) determine the value of a and the value of b .

(3)



Question 5 continued



Question 5 continued

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(Total for Question 5 is 12 marks)



6. Water is flowing into and out of a large tank.

Initially the tank contains 10 litres of water.

The rate of flow of the water is modelled so that

- there are V litres of water in the tank at time t minutes after the water begins to flow
- water enters the tank at a rate of $\left(3 - \frac{4}{1 + e^{0.8t}}\right)$ litres per minute
- water leaves the tank at a rate proportional to the volume of water remaining in the tank

Given that when $t = 0$ the volume of water in the tank is decreasing at a rate of 3 litres per minute, use the model to

(a) show that the volume of water in the tank at time t satisfies

$$\frac{dV}{dt} = 3 - \frac{4}{1 + e^{0.8t}} - 0.4V \quad (3)$$

(b) Determine $\frac{d}{dt}(\arctan e^{0.4t})$ (2)

Hence, by solving the differential equation from part (a),

(c) determine an equation for the volume of water in the tank at time t .

Give your answer in simplest form as $V = f(t)$ (6)

After 10 minutes, the volume of water in the tank was 8 litres.

(d) Evaluate the model in light of this information. (1)



Question 6 continued

Question 6 continued

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(Total for Question 6 is 12 marks)



7.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Explain why, for $n \in \mathbb{N}$

$$\sum_{r=1}^{2n} (-1)^r f(r) = \sum_{r=1}^n (f(2r) - f(2r-1))$$

for any function $f(r)$.

(2)

(b) Use the standard summation formulae to show that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{2n} r((-1)^r + 2r)^2 = n(2n+1)(8n^2 + 4n + 5)$$

(6)

(c) Hence evaluate

$$\sum_{r=14}^{50} r((-1)^r + 2r)^2$$

(4)



Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

8. A colony of small mammals is being studied.
In the study, the mammals are divided into 3 categories

N (newborns)	0 to less than 1 month old
J (juveniles)	1 to 3 months old
B (breeders)	over 3 months old

- (a) State one limitation of the model regarding the division into these categories.

(1)

A model for the population of the colony is given by the matrix equation

$$\begin{pmatrix} N_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} N_n \\ J_n \\ B_n \end{pmatrix}$$

where a and b are constants, and N_n , J_n and B_n are the respective numbers of the mammals in each category n months after the start of the study.

At the start of the study the colony has breeders only, with no newborns or juveniles.

According to the model, after 2 months the number of newborns is 48 and the number of juveniles is 40

- (b) (i) Determine the number of mammals in the colony at the start of the study.
(ii) Show that $a = 0.8$

(4)

- (c) Determine, in terms of b ,

$$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}^{-1}$$

(3)

Given that the model predicts approximately 1015 mammals **in total** at the start of a particular month, and approximately 596 **newborns**, 464 **juveniles** and 437 **breeders** at the start of the next month,

- (d) determine the value of b , giving your answer to 2 decimal places.

(3)

It is decided to monitor the number of **newborn** males and females as a part of the study. Assuming that 42% of newborns are male,

- (e) refine the matrix equation for the model to reflect this information, giving a reason for your answer.
(There is no need to estimate any unknown values for the refined model, but any known values should be made clear.)

(2)



Question 8 continued

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Question 8 continued



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(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

